Frequency Tracking and Variable Bandwidth for Line Noise Filtering without a Reference

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Abstract—**This paper presents a method for filtering line noise using an adaptive noise canceling (ANC) technique. This method effectively eliminates the sinusoidal contamination while achieving a narrower bandwidth than typical notch filters and without relying on the availability of a noise reference signal as ANC methods normally do. A sinusoidal reference is instead digitally generated and the filter efficiently tracks the power line frequency, which drifts around a known value. The filter's learning rate is also automatically adjusted to achieve faster and more accurate convergence and to control the filter's bandwidth. In this paper the focus of the discussion and the data will be electrocorticographic (ECoG) neural signals, but the presented technique is applicable to other recordings.**

Index Terms—adaptive filter, line noise, frequency tracking, variable bandwidth, neural signals, notch filter

I. Introduction

Contamination caused by line noise at 60 Hz or 50 Hz often presents a significant problem in the analysis of signal recordings. This is especially true in physiological recordings where the signal to noise ratio (SNR) can be low. Elimination of this contamination has been an active area of research, but many methods commonly implemented still fail to effectively eliminate the interference while minimizing distortion of the signal.

In neural signals for example, it is common practice to use a single fixed notch filter centered around the average power line frequency. The main problem with this approach is that power line frequency varies around its average frequency [1], so the notch must be wide enough to account for this variation. Increasing the notch width increases the possibility of also removing interesting physiological data. Other common approaches include low pass filtering below the power line frequency or doing a spectral analysis of the signal and ignoring those frequencies near the contamination. These techniques could also discard useful data.

The main objective of line noise removal is to eliminate the interference with minimum distortion or loss of the signal. This is best achieved by implementing a filter that is able to track the power line's slowly drifting frequency as in [2] and maintain a minimum filter bandwidth as in [3], [4]. In [5], an adaptive noise canceling (ANC) infinite impulse response (IIR) notch filter with varying poles and zeros was developed to accomplish both tasks, but this required manual adjustment of parameters and a reference input that was correlated with the line noise.

In some experimental setups, available hardware might make it difficult to simultaneously record a reference for power line interference. In this case a reference must be artificially generated and it becomes difficult for the ANC filter to outperform a standard notch filter in terms of noise rejection and signal retention. In order to maximize the potential of using an ANC filter for line noise removal without a recorded reference it is vital to be able to track the line noise frequency so that the generated reference can be as accurate as possible. It is also useful to have an adaptive learning rate to optimize the tradeoffs between convergence speed, accuracy, and tracking [4].

This paper provides methods for both frequency tracking and a variable bandwidth in an ANC filter. The methods add minimal complexity to the standard ANC filter configuration while proving highly effective in eliminating line noise and avoiding signal distortion.

II. Background

Adaptive filters have time-varying weights that adjust continually during adaptation to minimize the mean-square value of some error signal. In most cases the filter's goal is to converge to a state in which it imitates some unknown system. The ANC filter is a form of adaptive filter based largely on work done by Widrow [6] and has proven to be an effective means of removing noise that is correlated with some known reference signal [7]. In an ANC scheme the

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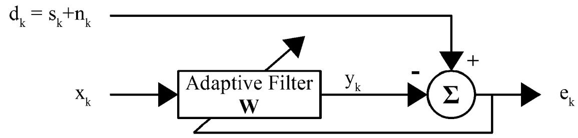

Figure 1: System design for an adaptive noise canceling (ANC) filter. *d* is the recorded signal that consists of the true signal *s* plus noise *n*. A reference correlated with *n* is given by *x*, and the filter coefficients ***W*** are adapted by minimizing the output *e* so that *y* most closely resembles *n* before being subtracted.

reference is filtered in order to minimize the difference between the filtered reference and the data. A typical ANC filter is given in Figure 1.

Here, s is the signal of interest before contamination and n is the additive noise, which is power line interference in this case. d is the actual recorded signal, x is the correlated reference signal for n, and e is both the error signal that feeds back to the adaptive filter and the output signal that should closely match s. The filter coefficients $\boldsymbol{W}$ will adapt so that y converges towards n. The sample index is denoted by k. One of the simplest and most commonly used convergence methods for adaptive filters is the least-mean-square (LMS) algorithm [8]. The LMS filter update equation is given in (1). The bound on μ, which is a step-size parameter, has been derived in [9] to ensure stability of the filter, where L is the filter length and σ_x^2 is the signal power of x.

$$W_{k+1}=W_k+2\mu e_k X_k\,,\quad 0<\mu<\frac{1}{L\sigma_x^2} \tag{1}$$

For line noise removal, x will be a sinusoid of the form in (2), where T is the sampling interval. In this special case, the ANC filter actually implements a 2nd order IIR notch filter centered at f_x, the frequency of the reference sinusoid. In [10] it was shown that the transfer function for an ANC filter in the case of a pure sinusoidal reference is given by (3). This is equivalent to a notch filter centered at f_x with bandwidth approximated by (4).

$$x_k=C\cos(2\pi f_x k T+\phi) \tag{2}$$

$$W(z)=\frac{z^2-2z\cos(2\pi f_x T)+1}{z^2-2\left(1-\frac{L\mu C^2}{4}\right)z\cos(2\pi f_x T)+\left(1-\frac{L\mu C^2}{2}\right)} \tag{3}$$

$$BW=\frac{L\mu C^2}{2\pi T}\ \ Hz \tag{4}$$

When given a sinusoidal reference, it is then known that the ANC technique with the LMS algorithm converges to a notch filter centered at the reference's frequency with a bandwidth that depends on μ. If the reference is an accurate model for the noise, then the ANC filter will also be able to easily track drifts in the frequency of the sinusoidal interference. This provides a far superior solution for line noise removal than a fixed notch filter. The challenge then becomes maintaining this performance in the absence of an accurate reference.

III. Methods

A. Signal Generation

An electrocorticographic (ECoG) signal simulator built in to Craniux, a software package for brain computer interface research, was used to generate eight channels of signals at 600 Hz [11]. These signals were free of line noise and were generated as pink noise with a $1/f$ power falloff to simulate an ECoG baseline signal [12] . Frequency band modulation was avoided, as risking different levels of modulation between experiments could give inaccurate analysis results. Higher modulation near the line noise would increase the relative importance of retaining the signal versus removing the noise. For analyzing the results of line noise removal, the general shape of the signal's spectrum is most important and this is maintained across experiments.

Simulated line noise n was added to s using (5), where i is the harmonic number and f_n is the fundamental line noise frequency. A_i was calculated to create a specified signal to noise ratio (SNR) between s and the 1st line noise harmonic, as shown in (6). P_s is the average power of s, which converges to a constant value. So as would be expected, the amplitude of the line noise did not vary significantly.

$$n=\sum_{i=1}^{M} A_i\cos(2\pi i f_n+\phi) \tag{5}$$

$$A_i=\frac{1}{2^{i-1}}\sqrt{\frac{P_s}{(1/2)\,10^{SNR_{db}/10}}} \tag{6}$$

The line noise fundamental frequency, f_n, was centered around 60 Hz and varied according to the Gauss-Markov process in (7) [4]. $\eta(\Delta_k)$ is a random sample from a zero-mean Gaussian distribution with variance σ_η^2. For further analysis, f_n was also controlled deterministically at times.

$$f_n(\Delta_{k+1})=f_n(\Delta_k)+\eta(\Delta_k) \tag{7}$$

B. Line Noise Removal

Since it has been shown that an ANC filter is capable of line noise removal if given an accurate reference signal for the sinusoidal interference, the difficulty lies in implementing an ANC scheme that does not rely on the reference signal. This was accomplished by taking advantage of the inherent structure of the ANC filter and making a few small changes to the LMS algorithm presented in Section II.

First, the normalized u given below in (8) will be used in place of μ. This is sometimes referred to as normalized LMS (NLMS). Any further reference to the learning rate will be referring to u. In addition to making the learning rate bounds easier to remember, the filter bandwidth given in (4) is further simplified by substituting in u. This results in a bandwidth given by (9), since for a sinusoid σ^2 is known to be $C^2/2$. The dependence on L and C has been removed.

$$\mu=\frac{u}{L\sigma_x^2}\,,\quad 0<u<1 \tag{8}$$

$$BW = \frac{u}{\pi T} \quad Hz \tag{9}$$

Next, a useful property of the ANC filter structure will be utilized to track the frequency of the sinusoidal noise. In Section II it was seen that in the ANC filter of Figure 1, the path from d to e when x is sinusoidal is a notch filter centered at f_x with a width that will here be given by (9). A corollary to this property is that the path from d to y is a bandpass filter matching the notch filter in center frequency and bandwidth [10].

This property means that if the true frequency of the line noise (f_n) drifts a sinusoid correlated to n, although attenuated, will be present in y. If no periodic components are present in s at a frequency near f_n, it is easy to track f_n with a technique as computationally simple as measuring time between zero crossings. Due to possible numeric error and noise introduced by the broadband signal components present near f_n, a smoothing filter such as the one in (10) was applied to the measured zero crossings. In (10), α is a forgetting factor set at 0.99, m is the index of the current zero crossing, and $\hat{f}_y$ is the estimated frequency.

$$\hat{f}_n(m) = \alpha * \hat{f}_y + (1-\alpha)\hat{f}_n(m-1) \tag{10}$$

Additionally, it was shown by (9) that the bandwidth of the notch filter, and thus the corresponding bandpass filter, is controlled by the learning rate u. This value can be automatically adjusted by $\Delta \hat{f}_n$, which allows the filter to increase its bandwidth when a change in f_n occurs. The increase in bandwidth helps maintain the elimination of the frequency-shifted line noise in e, and at the same time helps to decrease the attenuation of the line noise in y, causing the measurement of the zero crossings to be less affected by the broadband signal. As $\hat{f}_n$ approaches f_n, u decreases and the filter narrows around f_n. This decrease reduces the amount of the broadband signal eliminated in e and passed through to y, both improving the output and increasing the accuracy of $\hat{f}_n$, which allows u to decrease further. The process repeats in an iterative fashion and $\hat{f}_n$ converges towards f_n. To prevent instability, bounds were placed on u so that the filter's bandwidth remained between 0.2 and 2 Hz.

The end result is a notch filter that tracks f_n with a narrow bandwidth through a process that first increases bandwidth to locate a new value of f_n, then iteratively zooms back in to a narrow region around f_n. This process adds minimal computational cost to a standard ANC filter.

IV. Results and Discussion

Data was collected in the method described in Section III and the frequency tracking, variable bandwidth filtering technique presented in this paper was validated through comparison to more traditional line noise removal methods. The 2nd and 3rd line noise harmonics were also added to the data, and in the results presented here a corresponding filter for these harmonics was used for each technique. All results take the average across the eight channels of data.

For filtering techniques that have a fixed center frequency, their effectiveness only depends on the distance of the noise frequency from that center, not on how often and by how much the line noise frequency changes. Using the stochastic model given by (7) for these methods could give inconsistent results based on how far the line noise's mean frequency drifts from 60 Hz. So for comparison to traditional 2 Hz and 4 Hz notch filters, both standard methods for removing line noise in neural data, the frequency of the additive line noise was increased from 60 Hz by 0.1 Hz every 30 seconds to measure the resulting SNR at specific frequencies.

The SNR was measured between the true signal and the signal after having line noise added and filtered back out. In doing this measurement, both remaining line noise and signal distortion factor in to the SNR. As shown by Figure 2, the performance of the standard notch filters degraded as the frequency increased while the frequency tracking, variable bandwidth technique maintained a steady SNR that was higher even at 60 Hz, the ideal condition for the notch filters. Note that the SNR here also takes into account the time period during which the variable frequency filter is estimating and adjusting to the new frequency, and the small variance in the SNR is due to relatively short sample sizes.

Next, the effect of the filter's variable bandwidth was analyzed. The variable bandwidth is bounded by 0.2 and 2 Hz, so for comparison the filter was set up to first have a fixed bandwidth of 2 Hz, and then of 0.2 Hz. The performance of these settings for different changes in frequency is shown in Figure 3. With a fixed 0.2 Hz bandwidth the filter converged to the lowest MSE, but convergence time significantly increased as the magnitude of the changes in frequency increased. With a fixed 2 Hz bandwidth, the filter converged the quickest but not as accurately. With the variable bandwidth, the filter was able to converge quickly to a low MSE.

To obtain quantitative results for the filter's variable

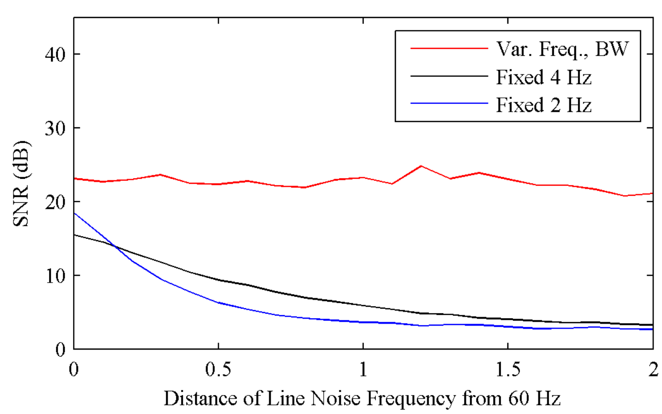

Figure 2: Performance of standard notch filters versus the variable frequency, variable bandwidth technique. The x-axis shows the distance of the line noise frequency from 60 Hz and the y-axis shows the SNR between the true signal and the filtered signal.

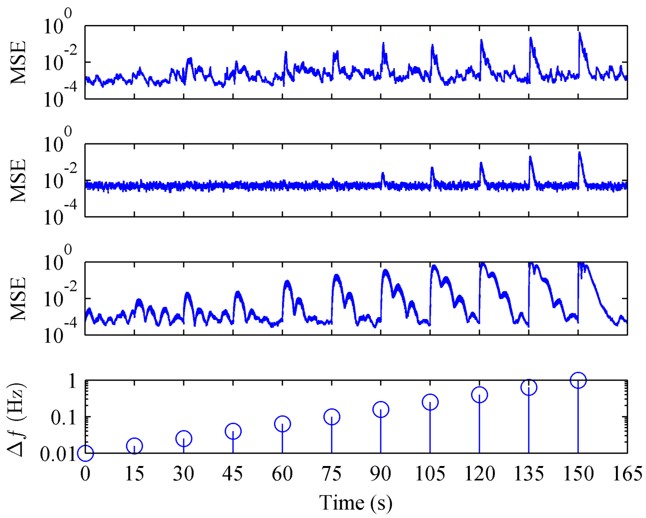

Figure 3: Frequency tracking capabilities with and without the variable bandwidth. The top three graphs represent the MSE between the true signal and filtered signal after the MSE was passed through a moving average filter of length ten to make the trend of convergence more visibly clear. (Top) variable bandwidth between 0.2 and 2 Hz, (Top Middle) bandwidth set at 2 Hz, and (Bottom Middle) bandwidth set at 0.2 Hz. The bottom graph represents the magnitude of changes in the line noise frequency that occurred at the corresponding time points in the top three graphs.

bandwidth, the stochastic model for line noise frequency given in (7) was employed with Δ_k = 2 seconds and σ_η = 0, 0.01, and 0.1. For σ_η = 0 the frequency remains at 60 Hz. The outcomes of these experiments are given in Table 1.

Table 1 presents results that are consistent with both Figure 2 and Figure 3. The first column of Table 1 again indicates that the variable bandwidth frequency tracking method outperforms traditional notch filters even under ideal circumstances for the notch filter. This is because the frequency estimate is able to converge closely to 60 Hz to eliminate the interference and then the bandwidth is narrowed to minimize distortion of the signal. Of the frequency tracking methods, the variable bandwidth method produces the highest average SNR across the 3 tested conditions, although at lower variances it is slightly outperformed by the 0.2 Hz bandwidth filter, which is able to maintain a tighter convergence since its bandwidth is unaffected by noise. At σ_η = 0.1, though, the 0.2 Hz bandwidth filter's performance drops significantly. The 2 Hz bandwidth filter performs well at all variances, but is unable to take advantage of the lower variances to converge more tightly around the line noise. The variable bandwidth method is able to increase performance at the lower variances and still maintain a good SNR at the highest variance.

V. Conclusions

The main objective of line noise removal is to eliminate the interference with minimum distortion to the signal. This is best achieved by implementing a filter that is able to track the power line's slowly drifting frequency and maintain a minimum filter bandwidth. A filter designed to accomplish this task in a computationally efficient manner without the use of a line noise reference signal was presented in this paper. Results show it is superior to traditional notch filters and able to perform effectively under varying conditions in the line noise. With the presented method, difficulties in tracking line noise frequency could arise if the SNR is too high, at which point noise removal is not as critical anyways, or if there is a periodic signal component near the line noise.

Further analysis of the presented filter should still be performed, including results from additional models for line noise frequency, a spectral analysis of the filter's performance, and the filter's behavior under different SNRs. Results could further be validated by a more in depth look at the accuracy of the frequency tracking method compared to more traditional and computationally costly methods such as spectral peak detection, which initial results have shown the presented method to outperform.

Table 1: SNR (dB) for Filtering Methods with Different Standard Deviations in the Line Noise Frequency Model

	$\sigma_\eta = 0$	$\sigma_\eta = 0.01$	$\sigma_\eta = 0.1$
Var. f_c, $0.2 \le BW \le 2$	24.4	23.2	18.4
Var. f_c, $BW = 2$	18.6	18.6	17.7
Var. f_c, $BW = 0.2$	27.5	24.5	10.9
2 Hz Notch	18.6	-	-
4 Hz Notch	15.3	-	-

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